

Predicting in-canopy velocity from free-stream velocity in obstructed environmental shear flows

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1. Introduction

One of the key features of environmental flows is the presence of permeable macro-roughness elements on the bed, such as vegetation, sediments or corals. While these obstructions differ in terms of morphology and size, they can be treated as hydrodynamically similar systems (Ghisalberti 2009). The drag of the obstructions retards the flow within the roughness leading to a local inflection in the mean velocity profile and hence Kelvin-Helmholtz-type shear instability at the canopy top. Given the fundamental role of the in-canopy flow for momentum and mass transport, a model is developed to predict the in-canopy flow velocity using both the (measured) free-stream velocity and the geometric features of the roughness.

2. Analytical framework

A double averaging approach is applied to remove spatial heterogeneities in the in-canopy flow. In the hypothesis of a stationary, uniform and infinitely wide flow driven by the free surface slope S , the horizontally-averaged longitudinal momentum balance can be expressed as:

$$\rho g S A_\varphi + \frac{\partial \tau}{\partial z} = f_x \quad (1)$$

with A_φ the roughness geometry function (volume of fluid divided by the total averaging volume), τ the fluid shear stress and f_x the local drag acting on the obstructions per unit volume. The drag is described by a quadratic law, $f_x = 0.5\rho C_d a \langle u \rangle_x^2$, with $C_d(z)$ the drag coefficient, $a(z)$ the frontal area of the obstruction per unit volume and $\langle u \rangle_x(z)$ the horizontally-averaged streamwise velocity. By integrating eq. 1 in the vertical direction from the bottom of the canopy to the flow depth δ , the total bed drag is given by the balance:

$$gS(\delta - h) + gSh\varphi = 0.5C_d\lambda_f U_c^2 \quad (2)$$

with h the obstruction height, $\varphi = \int_0^h A_\varphi/h dz$ the total bed porosity, $\lambda_f = \int_0^h a dz$ the frontal density and $U_c = \int_0^h \langle u \rangle_x/h dz$ the horizontally- and vertically-averaged in-canopy flow velocity. Eq. 1 assumes that C_d is constant in the vertical and that the contribution of the spatial fluctuation $\tilde{U} = \langle u \rangle_x - U_c$ to the drag is negligible. Since the shear velocity $u_* = \sqrt{gS(\delta - h)}$, the ratio (β) of the velocity at the free surface U_δ to U_c is given by (from eq. 2):

$$\beta = \frac{U_\delta}{U_c} = 1 + \frac{\Delta U}{u_*} \sqrt{\frac{C_d \lambda_f}{2(1+\varphi\frac{h}{\delta-h})}} \quad (3)$$

where $\Delta U = U_\delta - U_c$. Given the similarity of obstructed shear flows with mixing layers, Dupuis (2023) showed that the maximum shear stress in a mixing layer scales with the difference between the flow velocities at the edges of the mixing layers, $\tau_{max} \approx \rho k (U_2 - U_1)^2$ (where k describes the efficiency in momentum exchange). By taking, to first approximation, U_δ and U_c as analogous velocities to U_2 and U_1 for obstructed flows, and knowing that by definition $u_* = (\tau_{max}/\rho)^{1/2}$, eq. 3 can be expressed as:

$$\beta = \frac{U_\delta}{U_c} = 1 + K\sqrt{\alpha} \quad (4)$$



where $K = 1/\sqrt{k}$ and $\alpha = \sqrt{\frac{C_d \lambda_f}{2(1+\phi h/(\delta-h))}}$. Note that α depends only on the geometric features and on the drag coefficient of the obstruction. For high degrees of submergence, i.e. $h/\delta \ll 1$, β no longer depends on the obstruction porosity.

3. Preliminary validation of the framework

Eq. 4 was validated with a preliminary data analysis on selected double-averaged data taken in the laboratory from flows over gravel beds (Dey & Das 2012; Trevisson 2021), corals reefs (Pomeroy 2023) and rigid and flexible vegetation (Ghisalberti & Nepf 2006; Poggi et al. 2004). Coefficients of $C_d = 0.76$ and $C_d = 1$ were used for the gravel and corals/vegetation, respectively. Fig. 1 shows good collapse of the data. The value of $K \approx 8.1$ obtained from the data fitting in fig.1 corresponds to a value of $k \approx 0.0152 \pm 0.002$, close to values measured by Dupuis (2023) for shallow mixing layers. A larger amount of data covering also urban and terrestrial canopies will be subsequently included.

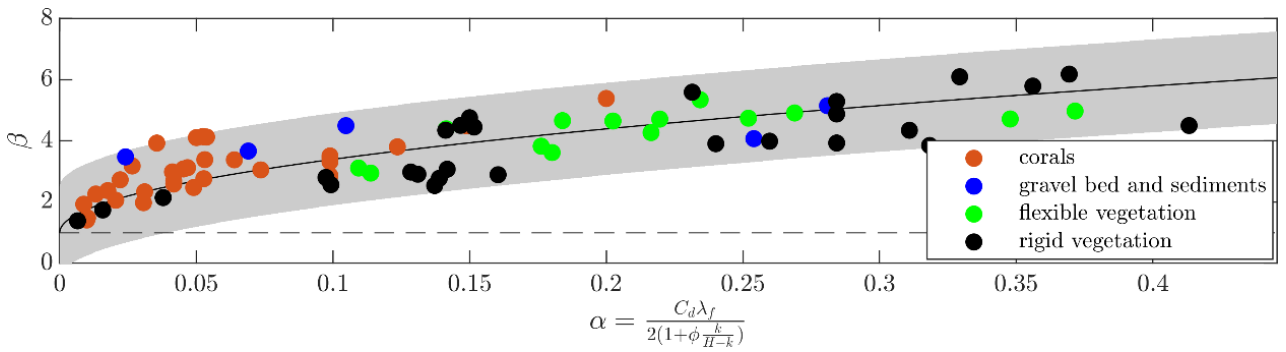


Figure 1. Performance of the predictive framework (eq. 4). The black line is the best fit with a 95 % prediction interval as shaded area and the dashed line $\beta = 1$

4. Conclusions

A predictive equation for the in-canopy velocity in obstructed shear flows was developed based on a mixing-layer analogy. Initial data analysis for published vegetated, coral- and gravel-covered beds suggests promise in using this model to predict in-canopy velocity, requiring only the free surface velocity, the total flow depth and the geometric features of the canopy as input.

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