

# Wall-normal scalar flux within a buoyant plume in a turbulent boundary layer

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## 1 Introduction

The Reynolds transport equation for the instantaneous concentration field,  $\tilde{C} = C + c$  (where  $C$  is mean and  $c$  is fluctuation), of a plume in a turbulent boundary layer, is not closed because the scalar fluxes,  $\overline{u_i c}$  are not known *a priori*. Here we denote the streamwise direction as  $x$  and the vertical direction as  $z$ . For a plume released within a turbulent boundary layer with the mean flow in the  $x$  direction, the wall-normal flux,  $\overline{w c}$  dominates the vertical spread of the plume. Typically, the gradient-diffusion hypothesis is used to model the fluxes, e.g.

$$-\overline{w c} = \kappa_z \cdot \partial C / \partial z. \quad (1)$$

It is well-known that the plume spread is different if the source is placed at different heights of the boundary layer. Hence it is not straightforward to specify distinct  $\kappa_i$  for each source height. The present study undertakes a study of the behaviour of  $\overline{w c}$  from experimental data.

The theoretical shape of  $\overline{w c}$  of an elevated source can be diagnosed by considering that  $\overline{w c} = 0$  at the plume centreline due to symmetry and far away outside the plume as concentration vanishes (Wyngaard, 2013). The location of the plume centreline,  $z_{cl}$ , is defined as the height of the maximum root mean square (RMS) concentration,  $\sigma_{c, \max}$ , in this abstract. Packets of concentration ( $+c$ ) moving upwards ( $+w$ ) and negative concentration fluctuations ( $-c$ ) moving downwards ( $-w$ ) result in  $\overline{w c} > 0$  above the plume centreline (Wyngaard, 2013). Similarly,  $\overline{w c} < 0$  below the plume centreline. Overall the profile has an s-shape.

The data used in this manuscript is an experimental study of buoyant plumes released from a point source in a turbulent boundary layer ( $Re_\tau \approx 1600$ ) at two different source heights. The density of a tracer gas mixture is varied to emulate the release of a neutrally, positively and negatively buoyant plume. Further details of the experiments can be found given in Talluru et al. (2017); Pang & Chauhan (2022).

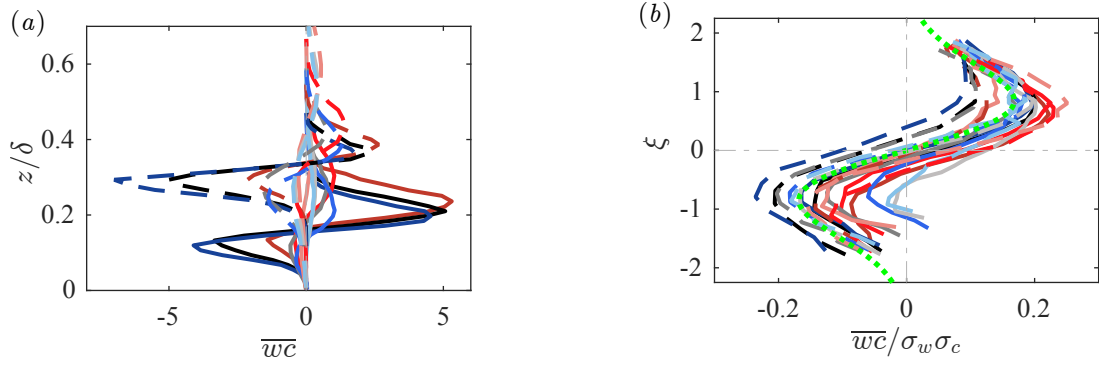
## 2 Normalisation of $\overline{w c}$ and non-dimensional eddy diffusivity

Figure 1(a) shows the distributions of  $\overline{w c}$  for two elevated sources as a function of  $z/\delta$ . The overall trend is similar to the prediction and that observed by past studies, where  $\overline{w c} > 0$  above the plume centreline and  $\overline{w c} < 0$  below. The overall magnitude of  $\overline{w c}$ , however, decreases with downstream distance as expected. Furthermore, the magnitude of  $\overline{w c}$  varies between the upper and lower halves of the plume since the vertical velocity fluctuation profile varies in the wall-normal direction. Since the profile exhibits a near-symmetrical behaviour about the plume centreline, the wall-normal distance relative to the location of the plume centreline,  $z_{CL}$ , is normalised using plume half-width,  $\delta_z$ , as  $\xi = (z - z_{cl})/\delta_z$  in figure 1(b). At the same time,  $\overline{w c}$  is normalised using the corresponding standard deviations,  $\sigma_w(z)$  and  $\sigma_c(z)$ . Upon normalisation, in figure 1(b), the similarity in shape and magnitude can be observed. The overall trend of the predicted reverse s-shape can be observed. It is noted that the  $y$ -axis intercept of  $\overline{w c}/\sigma_w \sigma_c$  is not necessarily zero, e.g. the dashed line in dark blue and the solid line in light blue in figure 2(b). This is possibly caused by the displacement zone (Kurbatskii & Yanenko, 1983), which will be investigated in a future study.

In the simplest gradient model as equation 1, the dimensional  $\overline{w c}$  is related to the gradient of the  $C$  profile. Alternatively,  $\overline{w c}$  can also be related to the gradient of the  $\sigma_c$  profile, since  $\sigma_c$  can also be described by the Gaussian or the reflected-Gaussian model, as in figure 1(a). The normalised profiles of  $\sigma_c$  are plotted in figure 2(a) and the derivative of standard Gaussian in figure 2(b). The derivative has a similar shape as the normalised  $\overline{w c}$  in figure 1(b), although the magnitude is different. Hence, a new relation of scalar flux is explored,

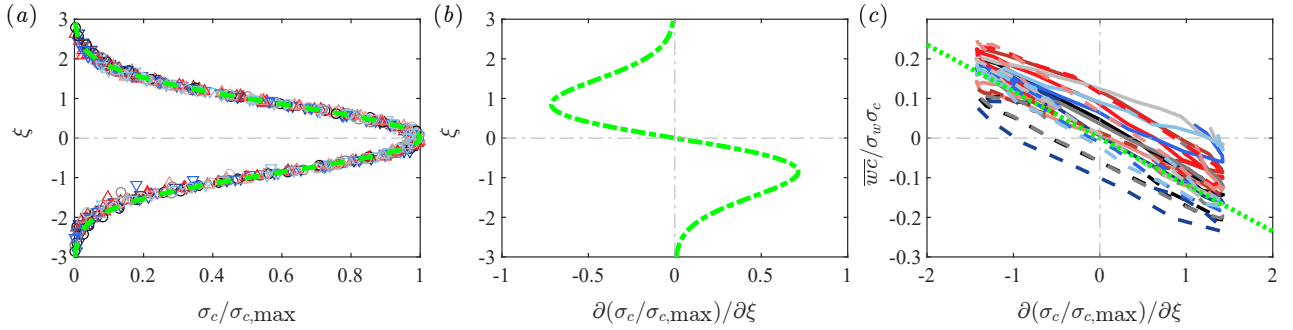
$$\frac{\overline{w c}}{\sigma_w \sigma_c} = \kappa_\sigma \frac{\partial(\sigma_c / \sigma_{c, \max})}{\partial \xi}, \quad (2)$$





**Figure 1.** (a) Wall-normal fluxes,  $\overline{w\overline{c}}$ , for source released at  $z/\delta = 0.16$  and  $0.32$  at three downstream locations. (b)  $\overline{w\overline{c}}/\sigma_w\sigma_c$  v.s. normalised wall-normal distance relative to the location of the plume centreline,  $\xi$ .

as plotted in figure 2(c). Note that  $\kappa_\sigma$  is a non-dimensional parameter and not the dimensional eddy-diffusivity. In the current study,  $\kappa_\sigma \approx -0.11$ . Consistency is representative of the two source elevations. The green dotted line in figure 2(c) describes the slope of all profiles well, though some scatter is observed.



**Figure 2.** (a) Normalised RMS concentration profiles. The green dashed line is the Gaussian model. (b) Derivative of the Gaussian model of  $\sigma_c$ . (c) Normalised  $\overline{w\overline{c}}$  v.s. derivative of the RMS of concentration. The green dotted line is the best fit of all the points with a slope of 0.11.

### 3 Conclusions

A novel method is employed to non-dimensionalise  $\overline{w\overline{c}}$ , where  $\overline{w\overline{c}}$  is divided by the local RMS of vertical velocity and RMS of concentration. The resulting normalised  $\overline{w\overline{c}}$  data collapse onto a single curve when plotted against the normalised distance to the plume centreline, in line with theoretical expectations. The normalised  $\overline{w\overline{c}}$  is then used to modify the traditional gradient diffusion model. A new non-dimensional parameter,  $\kappa_\sigma$ , is proposed to linearly relate the normalised  $\overline{w\overline{c}}$  and the gradient of normalised RMS of concentration. The model captures well the shape of normalised  $\overline{w\overline{c}}$  and shows good agreement with data. Unlike the traditional gradient-diffusion model,  $\kappa_\sigma$  remains constant with downstream distance, introducing an advantage. However, including second-order statistics in the Reynolds-averaged transport equation adds complexity.

### References

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